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A numerical example is given to show the effectiveness of proposed method.
March 31, 2008

Dr. Melvin Scott
Editor-in-Chief,
Applied Mathematics and Computation

Dear Dr. Melvin Scott

I am submitting a manuscript of paper entitled “State estimation for neural networks of neutral-type with interval time-varying delays” to be considered for publication in Applied Mathematics and Computation as a contributed paper. This is original work and has not been submitted for publication elsewhere.

Thank you for your anticipating efforts.

Yours sincerely,

Ju Hyun Park, Ph.D.
State estimation for neural networks of neutral-type with interval time-varying delays

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Abstract
In this paper, the design problem of state estimator for a class of neural networks of neutral type with interval time-varying delays is studied. The interval time-varying delay does not have constraint that its derivative is less than 1. The constraint is widely used to deal with time-varying delays in many papers. A delay-dependent linear matrix inequality (LMI) criterion for existence of the estimator is proposed by using Lyapunov method. The criterion can be easily solved by various convex optimization algorithms. A numerical example is given to show the effectiveness of proposed method.

Keywords: Neural networks, State estimator, neutral-type, Interval time-varying delay, LMI, Lyapunov method.

1. Introduction

During the last decades, considerable attention has been devoted to the study of artificial neural networks due to the fact that artificial neural networks can be applied to solve certain problems related to signal processing, static image treatment, image processing, pattern recognition, optimization and so on. Various models of neural networks such as Hopfield-type neural networks, cellular neural networks, Lotka-Volterra neural networks, Cohen-Grossberg neural networks, and bidirectional associative memory neural networks have been extensively investigated in the literature. For specific examples, see [1]-[5] and the references cited therein.

Time delay can be frequently encountered in the implementations of artificial neural networks, and its existence is often a source of oscillation and instability of the neural network. This motivates the study of stability of delayed neural networks. Therefore, many researchers have focused on the study for the stability analysis of delayed cellular neural networks during the last decade [6-19]. Recently, a special type of time delay, i.e., interval time-varying delay, is identified and investigated in real systems [20-23]. Interval time-varying delay is a time delay that varies in an interval in which the lower bound is not restricted to be zero. A typical real example of dynamic systems with interval time-varying delays is

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networked control systems [23].

On the other hand, due to the complicated dynamic properties of the neural cells in the real world, the existing neural network models in many cases cannot characterize the properties of a neural reaction process precisely. It is natural and important that systems will contain some information about the derivative of the past state to further describe and model the dynamics for such complex neural reactions [24-28]. In a practical manner, since the neuron states are not often fully available in the network outputs in many applications, the neuron state estimation problem is also important. Thus, the state estimation problem is investigated for several classes of neural network [29]-[31] in very recent years.

In this paper, we consider a class of neural networks with interval time-varying delays described by a nonlinear delay differential equation of neutral-type. The constraint that the time derivative of the interval time-varying delay is less than 1 is removed in this paper. The constraint is widely used in the literature to handle time-varying delay. The purpose of the paper is to estimate the neuron state via available output measurements such that the estimation error converges to zero. By constructing a suitable Lyapunov functional, a novel condition for existence of state estimator for the networks is given in terms of LMI. The advantage of the proposed approach is that resulting stability criterion can be used efficiently via existing numerical convex optimization algorithms such as the interior-point algorithms for solving LMIs [32].

Notation: Throughout the paper, $\mathbb{R}^n$ denotes the $n$ dimensional Euclidean space, and $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. $I$ denotes the identity matrix of appropriate dimensions. $\star$ represents the elements below the main diagonal of a symmetric block matrix. For symmetric matrices $X$ and $Y$, the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite, (respectively, nonnegative). diag{$\cdots$} denotes the block diagonal matrix.

2. Problem formulation

Consider the following class of delayed neural networks described by a nonlinear neutral delay differential equation:

\[
\begin{align*}
\dot{x}(t) &= -Ax(t) + W_1 f(x(t)) + W_2 g(x(t-h(t))) + V\dot{x}(t-\tau(t)) + b, \\
y(t) &= Cx(t) + z(t,x(t))
\end{align*}
\]

where $n$ denotes the number of neurons in a neural network, $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector, $f(x(t)) = [f_1(x_1(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$ and $g(x(t-h)) = [g_1(x_1(t-h(t))), \ldots, g_n(x_n(t-h(t)))]^T \in \mathbb{R}^n$ is the activation functions, $b = [b_1, \ldots, b_n]^T$ is the external bias at time $t$, $A = \text{diag}(a_i)$ is a positive diagonal matrix, $W_1 = (w_{ij}^1)_{n \times n}$, $W_2 = (w_{ij}^2)_{n \times n}$ and $V = (v_{ij})_{n \times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $z(t,x(t))$ is the neuron dependent nonlinear disturbances on the network output, and $h(t) > 0$ and $\tau(t) > 0$ correspond
to finite speed of axonal signal transmission delay satisfying the following:

\[ 0 \leq h_m \leq h(t) \leq h_M, \quad 0 \leq \tau(t) \leq \tau_M, \quad \dot{h}(t) \leq h_D, \quad \dot{\tau}(t) \leq \tau_D < 1. \]

Note that \( h(t) \) is the interval delay, and the activation function \( g(\cdot) \) may be defined as \( g(x(t - h(t))) \equiv f(x(t - h(t))) \) in many applications of neural networks.

Throughout the paper, it is assumed that the neurons activation functions \( f, \) and \( g \) and the disturbance function \( z \) are Lipschitz continuous:

\[
\begin{align*}
|f(x_1) - f(x_2)| &\leq |F(x_1 - x_2)|, \\
|g(x_1) - g(x_2)| &\leq |G(x_1 - x_2)|, \\
|z(x_1) - z(x_2)| &\leq |Z(x_1 - x_2)|
\end{align*}
\]

(2)

where \( F \in \mathbb{R}^{n \times n}, \ G \in \mathbb{R}^{n \times n} \) and \( Z \in \mathbb{R}^{n \times n} \) is the known constant matrices.

Then, the purpose of this paper is to present an efficient estimation algorithm to observe the neuron states from the available network output. For this end, the following full-order observer is proposed:

\[
\dot{\bar{x}}(t) = -A\bar{x}(t) + W_1 f(\bar{x}(t)) + W_2 g(\bar{x}(t) - h(t)) + V\dot{x}(t - \tau(t)) + b + K[y(t) - C\bar{x}(t) - z(t, \bar{x}(t))]
\]

(3)

where \( \bar{x}(t) \in \mathbb{R}^n \) is the estimation of the neuron state, and \( K \in \mathbb{R}^{n \times m} \) is the gain matrix of the estimator to be designed later.

Define the error state to be

\[
e(t) = x(t) - \bar{x}(t)
\]

(4)

and

\[
\phi(t) = f(x(t)) - f(\bar{x}(t)),
\]

\[
\varphi(t) = g(x(t)) - g(\bar{x}(t)),
\]

\[
\psi(t) = z(t, x(t)) - z(t, \bar{x}(t)).
\]

(5)

Then, the error dynamical system is expressed by

\[
\dot{e}(t) = -(A + KC)e(t) + W_1\phi(t) + W_2\varphi(t - h(t)) + V\dot{e}(t - \tau(t)) - K\psi(t).
\]

(6)

The following lemma will be used for deriving main result.

**Lemma 1.** [33] For any constant matrix \( \Sigma \in \mathbb{R}^{n \times n}, \Sigma = \Sigma^T > 0, \) scalar \( \gamma > 0, \) vector function \( \omega : [0, \gamma] \to \mathbb{R}^n \) such that the integrations concerned are well defined, then

\[
\left( \int_0^\gamma \omega(s)ds \right)^T \Sigma \left( \int_0^\gamma \omega(s)ds \right) \leq \gamma \int_0^\gamma \omega^T(s)\Sigma\omega(s)ds.
\]
3. Stability analysis

In this section, we derive a new delay-dependent criterion for asymptotic stability of the error system (6) using the Lyapunov method combining with LMI framework.

Then we have the following theorem.

**Theorem 1.** For given scalars $\tau_D$, $h_D$, $h_m$, $h_M$, and matrices $F, G, Z$, the error system (6) is globally asymptotically stable if there exist positive definite matrices $P, S, R, X, U_1, U_2, U_3, Q$, positive scalars $\alpha_i (i = 1, 2, 3)$, and any matrices $Y, M_i, N_i, \bar{M}_i, (i = 1, 2, \cdots, 9, a, b, c)$, satisfying the following LMI:

$$
\Sigma = \begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} & \Sigma_{18} & \Sigma_{19} & \Sigma_{1a} & \Sigma_{1b} & \Sigma_{1c} \\
* & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \Sigma_{26} & \Sigma_{27} & \Sigma_{28} & \Sigma_{29} & \Sigma_{2a} & \Sigma_{2b} & \Sigma_{2c} \\
* & * & \Sigma_{33} & \Sigma_{34} & \Sigma_{35} & \Sigma_{36} & \Sigma_{37} & \Sigma_{38} & \Sigma_{39} & \Sigma_{3a} & \Sigma_{3b} & \Sigma_{3c} \\
* & * & * & \Sigma_{44} & \Sigma_{45} & \Sigma_{46} & \Sigma_{47} & \Sigma_{48} & \Sigma_{49} & \Sigma_{4a} & \Sigma_{4b} & \Sigma_{4c} \\
* & * & * & * & \Sigma_{55} & \Sigma_{56} & \Sigma_{57} & \Sigma_{58} & \Sigma_{59} & \Sigma_{5a} & \Sigma_{5b} & \Sigma_{5c} \\
* & * & * & * & * & \Sigma_{66} & 0 & \Sigma_{68} & \Sigma_{69} & \Sigma_{6a} & 0 & 0 \\
* & * & * & * & * & \Sigma_{77} & \Sigma_{78} & \Sigma_{79} & \Sigma_{7a} & 0 & 0 \\
* & * & * & * & * & * & \Sigma_{88} & \Sigma_{89} & \Sigma_{8a} & \Sigma_{8b} & \Sigma_{8c} \\
* & * & * & * & * & * & * & \Sigma_{99} & \Sigma_{9a} & \Sigma_{9b} & \Sigma_{9c} \\
* & * & * & * & * & * & * & * & \Sigma_{aa} & \Sigma_{ab} & \Sigma_{ac} \\
* & * & * & * & * & * & * & * & * & \Sigma_{bb} & 0 \\
* & * & * & * & * & * & * & * & * & * & \Sigma_{cc}
\end{bmatrix} < 0
$$
with

$$
\Sigma_{11} = U_1 - PA - A^T P - YC - CTY^T + U_2 + GTXG + U_3 + \alpha_1 F^T F \\
+ \alpha_2Z^T Z + M_1 + M_1^T ,
$$

$$
\Sigma_{12} = M_2^T - M_1 + N_1, \Sigma_{13} = M_3^T - N_1 + \tilde{M}_1, \Sigma_{14} = M_4^T - M_1,
$$

$$
\Sigma_{15} = M_5^T - \alpha_1^T P - CTY^T, \Sigma_{16} = PV + M_6^T, \Sigma_{17} = PW_2 + M_7^T,
$$

$$
\Sigma_{18} = M_8^T - \tilde{M}_1, \Sigma_{19} = M_9^T - N_1, \Sigma_{1a} = M_a^T - \tilde{M}_1, \Sigma_{1b} = PW_1 + M_7^T,
$$

$$
\Sigma_{1c} = -Y + M_c^T, \Sigma_{22} = -U_1 - M_2 - M_2^T + N_2 + N_2^T,
$$

$$
\Sigma_{23} = -M_3^T + N_3^T - N_2 + \tilde{M}_2, \Sigma_{24} = -M_4^T - M_2 + N_4^T, \Sigma_{25} = -M_5^T + N_5^T,
$$

$$
\Sigma_{26} = -M_6^T + N_6^T, \Sigma_{27} = -M_7^T + N_7^T, \Sigma_{28} = -M_8^T + N_8^T - M_2,
$$

$$
\Sigma_{29} = -M_9^T + N_9^T - N_2, \Sigma_{2a} = -M_a^T + N_a^T - \tilde{M}_2, \Sigma_{2b} = -M_b^T + N_b^T,
$$

$$
\Sigma_{2c} = -M_c^T + N_c^T, \Sigma_{32} = -(1 - h_D)U_2 + \alpha_2^T G^T G - N_3 - N_3^T + \tilde{M}_3 + \tilde{M}_3^T,
$$

$$
\Sigma_{44} = -M_3 - N_4^T + \tilde{M}_4^T, \Sigma_{35} = -N_5^T + M_5^T, \Sigma_{36} = -N_b^T + M_b^T,
$$

$$
\Sigma_{37} = -N_7^T + \tilde{M}_7^T, \Sigma_{38} = -N_8^T + M_8^T - \tilde{M}_3, \Sigma_{39} = -N_9^T - N_3 + \tilde{M}_9^T,
$$

$$
\Sigma_{3a} = -N_a^T + \tilde{M}_a^T - \tilde{M}_3, \Sigma_{3b} = -N_b^T + \tilde{M}_b^T, \Sigma_{3c} = -N_c^T + \tilde{M}_c^T,
$$

5
\[ \Sigma_{44} = -Q - M_4 - M_4^T, \Sigma_{45} = -M_5^T, \Sigma_{46} = -M_6^T, \Sigma_{47} = -M_7^T, \]
\[ \Sigma_{48} = -M_8^T - M_4, \Sigma_{49} = -M_9^T - N_4, \Sigma_{4a} = -M_a^T - M_4, \Sigma_{4b} = -M_6^T, \]
\[ \Sigma_{4c} = -M_c^T, \Sigma_{55} = h_m^2 Q + (h_M - h_m)^2 S + R - 2P, \Sigma_{56} = PV, \]
\[ \Sigma_{57} = PW_2, \Sigma_{58} = -\bar{M}_5, \Sigma_{59} = -N_5, \Sigma_{5a} = -\bar{M}_5, \Sigma_{5b} = PW_1, \]
\[ \Sigma_{5c} = -Y, \Sigma_{66} = -(1 - \tau_D) R, \Sigma_{68} = -\bar{M}_6, \Sigma_{69} = -N_6, \]
\[ \Sigma_{6a} = -\bar{M}_6, \Sigma_{77} = -(1 - h_D) X - \alpha_2 I, \Sigma_{78} = -\bar{M}_7, \Sigma_{79} = -N_7, \]
\[ \Sigma_{7a} = -\bar{M}_7, \Sigma_{88} = -U_3 - \bar{M}_8 - M_b^T, \Sigma_{89} = -N_8 - M_b^T, \]
\[ \Sigma_{8a} = -\bar{M}_9^T - \bar{M}_8, \Sigma_{8b} = -\bar{M}_9^T, \Sigma_{8c} = -\bar{M}_9^T, \]
\[ \Sigma_{9g} = -S - N_9 - N_c^T, \Sigma_{9a} = -N_a^T - M_9, \Sigma_{9b} = -N_b^T, \]
\[ \Sigma_{9c} = -N_c^T, \Sigma_{aa} = -S - M_a - M_a^T, \Sigma_{ab} = -\bar{M}_5^T, \]
\[ \Sigma_{ac} = -\bar{M}_c^T, \Sigma_{bb} = -\alpha_1 I, \Sigma_{cc} = -\alpha_3 I. \]

Then, the gain matrix \( K \) of the state estimator \( (3) \) is given by
\[ K = P^{-1} Y. \]

**Proof.** Let us consider the following Lyapunov functional candidate:
\[ V = \sum_{i=1}^{8} V_i \quad (7) \]

where
\[ V_1 = e^T(t)P e(t), \quad V_2 = \int_{t-h_m}^{t} e^T(s)U_1 e(s) ds, \]
\[ V_3 = \int_{t-h(t)}^{t} e^T(s)U_2 e(s) ds, \quad V_4 = h_m \int_{t-h_m}^{t} (s - (t - h_m)) e^T(s)Q e(s) ds, \]
\[ V_5 = (h_M - h_m) \int_{t-h_M}^{t} (s - (t - h_M)) e^T(s)S e(s) ds, \quad V_6 = \int_{t-h(t)}^{t} \varphi^T(s) X \varphi(s) ds, \]
\[ V_7 = \int_{t-\tau(t)}^{t} \dot{e}(s)R e(s) ds, \quad V_8 = \int_{t-h_m}^{t} e^T(s)U_3 e(s) ds. \]

First, differentiating \( V_1 \) leads to
\[ \dot{V}_1 = 2e^T(t)P \left( -(A + KC) e(t) + W_1 \dot{e}(t) + W_2 \psi(t - h(t)) + V \dot{e}(t - \tau(t)) - K \psi(t) \right). \quad (8) \]
Next, by differentiating $V_i(i = 2, 3, \cdots, 5)$, we have

\[
\dot{V}_2 = e^T(t) U_1 e(t) - e^T(t - h_m) U_1 y(t - h_m),
\]

\[
\dot{V}_3 \leq e^T(t) U_2 e(t) - (1 - h_D) e^T(t - h(t)) U_2 y(t - h(t)),
\]

\[
\dot{V}_4 = \dot{h}_m^2 e^T(t) \dot{q}(t) - \dot{h}_m \int_{t - h_m}^{t} \dot{e}^T(s) Q \dot{e}(s) ds,
\]

\[
\leq \dot{h}_m^2 e^T(t) \dot{q}(t) - \left( \int_{t - h_m}^{t} \dot{e}^T(s) ds \right) Q \left( \int_{t - h_m}^{t} \dot{e}(s) ds \right)
\]

\[
\dot{V}_5 = (h_M - \dot{h}_m)^2 \dot{e}^T(t) \dot{e}(t) - (h_M - \dot{h}_m) \int_{t - h_M}^{t - h(t)} \dot{e}^T(s) \dot{e}(s) ds
\]

\[
= (h_M - \dot{h}_m)^2 \dot{e}^T(t) \dot{e}(t) - (h_M - \dot{h}_m) \int_{t - h(t)}^{t - h_M} \dot{e}^T(s) \dot{e}(s) ds
\]

\[
-(h_M - \dot{h}_m) \int_{t - h_M}^{t - h(t)} \dot{e}^T(s) \dot{e}(s) ds
\]

\[
\leq (h_M - \dot{h}_m)^2 \dot{e}^T(t) \dot{e}(t) - \left( \int_{t - h(t)}^{t - h_M} \dot{e}^T(s) ds \right) S \left( \int_{t - h(t)}^{t - h_M} \dot{e}(s) ds \right)
\]

\[
- \left( \int_{t - h_M}^{t - h(t)} \dot{e}^T(s) ds \right) S \left( \int_{t - h_M}^{t - h(t)} \dot{e}(s) ds \right).
\]

Finally, we obtain

\[
\dot{V}_6 \leq \varphi^T(t) X \varphi(t) - (1 - h_D) \varphi^T(t - h(t)) X \varphi(t - h(t)),
\]

\[
\leq e^T(t) G^T X e(t) - (1 - h_D) \varphi^T(t - h(t)) X \varphi(t - h(t)),
\]

\[
\dot{V}_7 \leq \dot{e}^T(t) \dot{r}(t) - (1 - \tau_D) \dot{e}^T(t - \tau(t)) \dot{r}(t - \tau(t)),
\]

\[
\dot{V}_8 = e^T(t) U_3 e(t) - e^T(t - h_M) U_3 y(t - h_M).
\]

From Eqs. (2) and (5), it is clear that

\[
\varphi^T(t) \varphi(t) = |f(x(t) - f(\bar{x}(t))|^2 \leq |Fe(t)|^2 = e^T(t) F^T Fe(t),
\]

\[
\varphi^T(t) \varphi(t) = |g(x(t) - g(\bar{x}(t))|^2 \leq |Ge(t)|^2 = e^T(t) G^T Ge(t),
\]

\[
\psi^T(t) \psi(t) = |z(x(t) - z(\bar{x}(t))|^2 \leq |Ze(t)|^2 = e^T(t) Z^T Ze(t). \]
Then, for positive scalars \( \alpha_i (i = 1, 2, 3) \), we have

\[
\begin{align*}
\alpha_1 [e(t)^T F^T Fe(t) - \phi^T (t) \phi(t)] & \geq 0, \\
\alpha_2 [e^T (t - h(t)) G^T Ge(t - h(t)) - \varphi^T (t - h(t)) \varphi(t - h(t))] & \geq 0, \\
\alpha_3 [e^T (t) Z^T Z e(t) - \psi^T (t) \psi(t)] & \geq 0.
\end{align*}
\]

(17)

Also, for any constant matrices \( N_i (i = 1, \cdots, 9, a, b, c) \), \( M_i (i = 1, \cdots, 9, a, b, c) \), \( \bar{M}_i (i = 1, \cdots, 9, a, b, c) \) with appropriate dimensions, the following zero equations hold:

\[
\begin{align*}
2 \left[ e^T (t) M_1 + e^T (t - h_m) M_2 + e^T (t - h(t)) M_3 + \left( \int_{t-h_m}^t \dot{e}(s) ds \right)^T M_4 + \dot{e}^T (t) M_5 \\
+ \dot{e}^T (t - \tau(t)) M_6 + \varphi^T (t - h(t)) M_7 + e^T (t - h_M) M_8 + \left( \int_{t-h_M}^{t-h_m} \dot{e}(s) ds \right)^T M_9 \\
+ \left( \int_{t-h_M}^{t-h(t)} \dot{e}(s) ds \right)^T M_a + \phi^T (t) M_b + \psi^T (t) M_c \right] \\
\times \left[ e(t) - e(t - h_m) - \int_{t-h_m}^t \dot{e}(s) ds \right] & = 0,
\end{align*}
\]

(18)

\[
\begin{align*}
2 \left[ e^T (t) N_1 + e^T (t - h_m) N_2 + e^T (t - h(t)) N_3 + \left( \int_{t-h_m}^t \dot{e}(s) ds \right)^T N_4 + \dot{e}^T (t) N_5 \\
+ \dot{e}^T (t - \tau(t)) N_6 + \varphi^T (t - h(t)) N_7 + e^T (t - h_M) N_8 + \left( \int_{t-h_M}^{t-h_m} \dot{e}(s) ds \right)^T N_9 \\
+ \left( \int_{t-h_M}^{t-h(t)} \dot{e}(s) ds \right)^T N_a + \phi^T (t) N_b + \psi^T (t) N_c \right] \\
\times \left[ e(t - h_m) - e(t - h(t)) - \int_{t-h(t)}^{t-h_m} \dot{e}(s) ds \right] & = 0,
\end{align*}
\]

(19)
\[
2 \left[ e^T(t) \bar{M}_1 + e^T(t - h_m) \bar{M}_2 + e^T(t - h(t)) \bar{M}_3 + \left( \int_{t-h_m}^t \dot{e}(s) \, ds \right)^T \bar{M}_4 + \dot{e}^T(t) \bar{M}_5 \right.
\]
\[
+ \dot{e}^T(t - \tau(t)) \bar{M}_6 + \varphi^T(t - h(t)) \bar{M}_7 + e^T(t - h_M) \bar{M}_8 + \left( \int_{t-h(t)}^{t-h_M} \dot{e}(s) \, ds \right)^T \bar{M}_9
\]
\[
+ \left( \int_{t-h_M}^{t-h(t)} \dot{e}(s) \, ds \right)^T \bar{M}_a + \phi^T(t) \bar{M}_b + \psi^T(t) \bar{M}_c \right]
\]
\[
\times \left[ e(t - h(t)) - e(t - h_M) - \int_{t-h_M}^{t-h(t)} \dot{e}(s) \, ds \right] = 0,
\]

(20)

Note that the four zero equations above are used to get less conservative criterion by introducing free variables.

Here, let us define an augmented vector \( \zeta \):
\[
\zeta^T = \begin{bmatrix}
e^T(t) & e^T(t - h_m) & e^T(t - h(t)) & \left( \int_{t-h_m}^t \dot{e}(s) \, ds \right)^T & \dot{e}^T(t) & \dot{e}^T(t - \tau(t)) \\
\varphi^T(t - h(t)) & e^T(t - h_M) & \left( \int_{t-h_M}^{t-h(t)} \dot{e}(s) \, ds \right)^T & \left( \int_{t-h_M}^{t-h(t)} \dot{e}(s) \, ds \right)^T & \phi^T(t) & \psi^T(t)
\end{bmatrix},
\]

and let us define a new variable \( Y \) as \( Y = PK \).

Then, using Eqs. (17) and (18)-(21), a new bound of \( \dot{V} \) is given by
\[
\dot{V} \leq \zeta^T \Sigma \zeta,
\]

(22)

where the matrix \( \Sigma \) is defined in Theorem 1.

Hence if the condition given in Eq. (7) holds, the system (6) is asymptotically stable by the Lyapunov theory, i.e., the state estimator (3) tracks the system (1). This completes our proof.

\[ \blacksquare \]

**Remark 1.** The criterion given in Theorem 1 is delay-dependent. It is well known that the delay-dependent criteria are less conservative than delay-independent criteria when the delay is small. The solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, which is a convex optimization problem [32]. In this paper, we utilize Matlab’s LMI Control Toolbox [34] which implements interior-point algorithm. This algorithm is significantly faster than classical convex optimization algorithms [32].

**Remark 2.** In order to handle with interval time-varying delays \( h(t) \) in many papers, the constraint that the derivative of \( h(t) \) is less than 1 is needed for deriving certain stability criteria. However, the
constraint is not used in this work.

Now, we give a numerical example to illustrate our main result and its usefulness.

**Example 1.** Consider the delayed neural network with the following parameters:

\[
A = \text{diag}\{3, 3, 2\}, \quad W_1 = \begin{bmatrix} 1 & 0.1 & 0 \\ 0.1 & 0.3 & 0.2 \\ 0.2 & 0.1 & 1 \end{bmatrix},
\]

\[
W_2 = \begin{bmatrix} 0.3 & 1 & 0.2 \\ 0.1 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.5 \end{bmatrix}, \quad V = \begin{bmatrix} 0.2 & 0.05 & 0.1 \\ 0.05 & 0.1 & 0.05 \\ 0.05 & 0.1 & 0.2 \end{bmatrix}, \quad C = I,
\]

\[
f(x) = 0.5 \sin(x(t)), \quad g(x(t - h(t))) = 0.5 \sin(x(t - h(t))), \quad z(x(t)) = 0.5 \sin(4x(t)),
\]

\[
J = \begin{bmatrix} \sin(4t) + 0.005t^2 \\ -\sin(4t) - 0.004t^2 \\ 1.2\sin(4t) + 0.01t^2 \end{bmatrix}, \quad x(0) = [-2 \ 2 \ -0.5]^T.
\]

From the functions \(f, g, z\), it is easy to see that \(F = 0.5I, G = 0.5I, Z = 0.5I\).

Now, let us apply Theorem 1 to above system for several cases.

First, in case of time-invariant delays, \((h_D = 0, \tau_D = 0)\), the LMI given in Theorem 1 is feasible for any delay \(h > 0\). For example, when the system parameters are set to \((h(t) = 5, \tau(t) = 0.3)\), the gain matrix, \(K\), of state estimation is computed as:

\[
K = \begin{bmatrix} 1.3566 & -0.0009 & -0.0232 \\ 0.0033 & 1.3447 & -0.0255 \\ -0.0180 & -0.0178 & 0.7438 \end{bmatrix}.
\] (23)

When it comes to apply the state estimator with \(K\) obtained in (23) and its zero initial condition, the simulation results are shown in Fig. 1. From the figure, one can see that the responses of the state estimators track to true states quickly.

Next, in case of \((h_m = 0, h_M = 10.0, h_D = 0.9, \tau_D = 0.3)\), the gain matrix \(K\) of state estimation is:

\[
K = \begin{bmatrix} 1.4572 & -0.0431 & -0.0620 \\ 0.0192 & 1.3419 & -0.0390 \\ -0.0379 & -0.1306 & 0.7565 \end{bmatrix}.
\] (24)

For \((h_m = 0.5, h_M = 1.5, h_D = 1.5, \tau_D = 0.3)\), we have

\[
K = \begin{bmatrix} 1.6802 & 0.0528 & -0.0080 \\ 0.0631 & 1.3888 & -0.0151 \\ -0.0321 & -0.0264 & 0.9398 \end{bmatrix}.
\] (25)
4. Concluding remarks

In this paper, we have dealt with the problem of state estimation for neural networks of neutral-type with interval time-varying delay for the first time. A linear matrix inequality (LMI) approach has been developed to solve the problem addressed. Finally, a numerical examples have been illustrated to demonstrate the usefulness of the main results.

References


Figure 1: The true state $x_i(t)$ and its estimate $\bar{x}_i(t)$